An Appointment Overbooking Model To Improve Client Access and Provider Productivity

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New Challenges in Service Operations  
POMS College of Service Operations and EurOMA Conference  

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Appointment Scheduling
Appointment Waiting

"When I arrived I was cleanshaven."
Appointment Services

- Types of services
  - Medical care & mental health clinics
  - Dentists and medical specialists
  - Government offices; law offices
  - Counseling & admissions offices
  - Retail (tax help, salons, …)

We call these “appointment services”
- Where “providers” in “offices” serve “clients”
The “No-Show” Problem

- Research motivated by a outpatient mental health clinic in Denver, CO
  - 16 daily appointments / clinician
  - 30% no show rate

- Office no-show rates vary from 0-80%
  - <10% Brahimi & Worthington (1991); Warden (1995)
  - 10-30% Barron (1980)
  - 3-80% Rust et al. (1995)
Possible Solutions

- Sending clients reminder cards

- Call clients to remind them of appointments

- Providing public transport information
  - Bean & Talaga (1995)

- Overbooking has not been closely examined as a possible response
  - Widely used other businesses (e.g., airlines)
Literature

- **Blanco White & Pike (1964)**
  - Appointment systems in out-patients’ clinics and effect of patients’ unpunctuality. *Medical Care* 2(3), 133-145

- **Vissers (1979)**
  - Selecting a suitable appointment system in an outpatient setting. *Medical Care*, 17(12), 1207-1220

- **Cayirli & Veral (2003)**

- **LaGanga & Lawrence (2007a)**
  - Clinic overbooking to improve patient access and increase provider productivity. *Decision Sciences*, 38(2).

- **LaGanga & Lawrence (2007b)**
Appointment Scheduling and Overbooking Model

![Bar Chart]

Number Waiting (k)

Probability

0% 10% 20%

0 1 2 3 4 5 6 7 8 9 10 11 12
How to Handle No-Shows?

- How to balance competing goals?
  - Provide better client access
  - Minimize client waiting
  - Minimize office overtime
  - Maximize provider productivity

- How to measure a “good” policy?
  - Is this a monetary problem?
  - A service problem?
Overbooking Utility Model

- Maximize office “utility”
- Trade-off
  - Client access (number of clients seen)
  - Average client waiting times
  - Expected office overtime
- Note that provider productivity is implicit in this model
Assumptions

- Clients “show” on time with probability $\sigma$
- Client service times deterministic
  - No variability
- Clients serviced by assigned provider
- Office accrues
  - Benefits for serving additional clients
  - Penalties for keeping clients waiting
  - Penalties for office overtime
Probability that $a_j$ Clients Arrive

- Arrivals are binomially distributed
  - $s_j$ clients scheduled for appt slot $j$
  - Probability of a client showing is $\sigma$
  - $a_j \leq s_j$ clients show for appointment

$$f(a_j; s_j, \sigma) = \binom{s_j}{a_j} \sigma^a_j (1-\sigma)^{s_j-a_j} = \frac{s_j!}{a_j!(s_j-a_j)!} \sigma^a_j (1-\sigma)^{s_j-a_j}$$
Arrival Distribution Example

3 clients scheduled; 50% show rate

$N = 162, \ \sigma = 50\%, \ slot \ j = 12, (\omega, \tau) = (0.5, 1.0) \ linear$
Probability of $k$ clients Waiting

\[ \theta_{j+1,k} = \theta_{j,0} \alpha_{j+1,k} + \sum_{i=0}^{k} \theta_{j,i+1} \alpha_{j+1,k-i} \]

- $\alpha_{jk} = \text{probability of } k \text{ clients arriving for service at the start of appointment slot } j$
- $\theta_{jk} = \text{probability of } k \text{ clients waiting for service at start of appointment slot } j$
Number Waiting Example

Appointment slot 12; 3 clients scheduled

\[ N = 16, \sigma = 50\%, \text{ slot } j = 12, (\omega, \tau) = (0.5, 1.0) \text{ linear} \]
Relative Benefits and Penalties

- \( \pi \) = Benefit of seeing additional client
- \( \omega \) = Penalty for client waiting
- \( \tau \) = Penalty for office overtime
- Numéraire of \( \pi \), \( \omega \), and \( \tau \) doesn’t matter
  - Ratios (relative importance) are important
- Allow both linear and quadratic costs
Linear & Quadratic Costs

- Model allows 2\textsuperscript{nd} order polynomials
  - Results not reported in this paper
Linear & Quadratic Objectives

- **Linear Utility Function**

\[
\hat{U}^L(S) = \pi \hat{A} - \frac{\omega}{\hat{A}} \left( \sum_{j=1}^{N} \sum_{k} k \theta_{jk} + \sum_{k} \sum_{i=1}^{k} i \theta_{N+1,k} \right) - \tau \sum_{k} k \theta_{N+1,k}
\]

- **Quadratic Utility Function**

\[
\hat{U}^Q(S) = \pi \hat{A} - \frac{\omega}{\hat{A}} \left( \sum_{j=1}^{N} \sum_{k} (2k-1) \theta_{jk} + \sum_{k} \sum_{i=1}^{k} i^2 \theta_{N+1,k} \right) - \tau \sum_{k} k^2 \theta_{N+1,k}
\]
Solution Methodology

1. Gradient search
   - Increment/decrement appointments scheduled in each slot
   - Choose the single change which provides the greatest improvement in utility
   - Iterate until no further improvement found

2. Pairwise interchange
   - Exchange appointments scheduled in all appointment slot pairs
   - Choose the single swap which provides the greatest improvement in utility
   - Iterate until no further improvement found
3. Computational Results

![Bar Chart]

- X-axis: Appointment Slot
- Y-axis: Number of Appointments

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<th>Appointment Slot</th>
<th>Number of Appointments</th>
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Example Schedules

- 180 example problems solved
- Office sizes
  - $N = \{4, 8, 12, 16, 20, 24\}$
- Show rates
  - $\sigma = \{90\%, 80\%, \ldots, 50\%\}$
- Benefit of additional client
  - $\pi = 1.0$
- Waiting / overtime costs
  - $(\omega, \tau) = (1.0, 1.0) (0.5, 1.5) (1.5, 1.5)$
- Linear and quadratic cost functions
Example Schedules (1/3)

6A. $N=4$, $\sigma = 0.8$
$(\omega, \tau) = (0.5, 1.5)$ quadratic

6B. $N=8$, $\sigma = 0.5$
$(\omega, \tau) = (1.0, 1.0)$ linear

- **Front-loading**
  - Bailey (1952)

- **Double-booking**
  - Welch & Bailey (1952)
Example Schedules (2/3)

6C. $N = 12, \sigma = 0.7$
$(\omega, \tau) = (1.0, 1.0)$ quadratic

6D. $N = 16, \sigma = 0.5$
$(\omega, \tau) = (1.0, 1.0)$ linear

- Wave schedule
  - Baum (2001)

- Front-loading + double-booking
Example Schedules (3/3)

6E. $N = 20, \sigma = 0.8$
$(\omega, \tau) = (0.5, 1.5)$ linear

6F. $N = 24, \sigma = 0.5$
$(\omega, \tau) = (0.5, 1.5)$ quadratic

- Waves with increasing period
- Front-loading + double-booking + erratic waves
Appointments Overbooked

1A. Overbooking vs. office size $N$

1B. Overbooking vs. show rate $\sigma$


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2A. Utility improvement vs. office size $N$

2B. Utility improvement vs. show rate $\sigma$
Without overbooking, provider productivity is equal to the show rate $\sigma$. 

3A. Productivity vs. office size $N$ 

3B. Productivity vs. show rate $\sigma$
Expected Waiting & Overtime

4A. Expected waiting vs. office size $N$

4B. Expected waiting vs. show rate $\sigma$

4C. Expected overtime vs. office size $N$

4D. Expected overtime vs. show rate $\sigma$
Overbooking Patterns

5A. Linear costs

5B. Quadratic costs
Managerial Implications

- Overbooking (OB)
  - Improves customer service (serve more)
  - Increases provider utilization
  - Increases expected client wait times
  - Increases expected clinic overtime

- OB patterns are problem specific
  - Unlikely simple rules will satisfice
  - Need optimal or near-optimal schedules
Contributions of Research

- Demonstrate benefits of appointment overbooking
- Analytic model of appointment scheduling with overbooking
  - Maximize utility
  - Balance service, waiting, and overtime
  - Linear and quadratic cost functions
- Fast and effective heuristic solutions
- Previous literature shown to be special cases of our analytic model
Future Extensions

- Stochastic service times
- Service times vary by service type
- Show rates vary by time of day
- Appointments scheduled at any time
  - Not just at start of appointment slot
- Walk-ins
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